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Companding based PAPR Reduction Technique in OFDM Systems

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***Abstract* – Orthogonal Frequency Division Multi-plexing (OFDM) have been extensively applied in wireless communication systems. One of the main drawbacks of orthogonal frequency division multiplexing is the high peak-to-average power ratio at the transmitter. The uniformly distributed nonlinear companding techniques reduce the high PAPR of OFDM signals. The uniformly distributed companding technique cannot satisfy the different performance requirement for the systems. In this paper proposed, a novel uniformly distributed companding scheme that transforms the OFDM signal into trapezium distribution. The uniformly distributed companding schemes efficiently reduce the PAPR and improve Bit Error Rate (BER) for OFDM systems. Finally, Simulation results consider a baseband OFDM system with Additive White Gaussian Noise (AWGN) and multipath channels show that the proposed scheme provides trade-off between the PAPR reduction and the BER.**

***Keywords* – Companding, Orthogonal Frequency Division multiplexing (OFDM), Peak-to-Average Power Ratio (PAPR).**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) techniques allows for achieving high speed transmission in wireless channel. OFDM has a number of applications including Digital Audio Broadcasting (DAB), the ETSI HIPERLAN/2 standard, Terrestrial Digital Video Broadcasting (DAB-T), the IEEE 802.11a standard for Wireless Local Area Network (WLAN), the IEEE 802.16d standard for wireless Metropolitan Area Network. OFDM provides greater immunity to multipath fading and impulse noise, and removes the need for equalizers.

The main drawbacks of the OFDM system have very high Peak-to-average power ratio at the transmitter [1]. When the OFDM signals are transmitted with high PAPR through high power amplifier or Digital to analog convertor (DAC), a high peak signal arises out-of-band energy and in-band distortion. These conditions affect the performance of OFDM systems. There are many techniques to reduce PAPR of the OFDM systems, such as amplitude clipping [2], clipping and filtering [3], window shaping [4], block coding [5,6], active constellation extension (ACE) [7], tone reservation and tone injection [8], phase optimisation [9], interleaving [10], partial transmit sequence [11], selective mapping [12,13]. These techniques divided into two categories.

The first category is to reduce the high PAPR signals before multicarrier modulation, such as coding [5,6], selective mapping [12,13] and partial transmit sequence [11]. The authors maintain the original Bit Error Rate (BER) of OFDM system but require large number of computational complexity. The other category is to reduce PAPR with the signals after multicarrier modulation, such as clipping [3], companding techniques [14,15]. The clipping technique is simple and most widely used for reducing the PAPR of OFDM signals. The clipping noise becomes very significant with high modulators order, which makes companding more suitable for high data rates application and affect the system performance [16]. The μ-law companding scheme shows the better performance than clipping, but the PAPR is reduced at the expense of an increase in the average power. The nonlinear companding scheme such as expontial companding scheme [17], transforms the amplitude of the original OFDM signals without changing the average power. In addition, the piecewise nonlinear companding scheme [15] compare with the exponential companding scheme [17] and offers [15] a trade-off between BER performance and PAPR reduction by adjusting two parameter and its companding function is a piecewise function with three pieces.

In this work proposes a new nonlinear companding scheme to transforms the original OFDM signals into trapezium distribution by regulating a parameter that shows the trapezium distribution and companding function is continuous function. Furthermore, the theoretical CCDF of PAPR are derived and the attenuation factor caused by the nonlinear operation. The simulation results compared with uniformly-distributed scheme and the piecewise nonlinear companding scheme.

The rest of this paper is organized as follows. Section II introduces the PAPR problem of the OFDM system, the uniformly-distributed companding scheme and the theoretical analysis of the CCDF of the PAPR. In Section III shows the Proposed Companding Technique. In Section IV introduces Criteria for Selection of PAPR reduction Techniques. Finally conclusion for this work.

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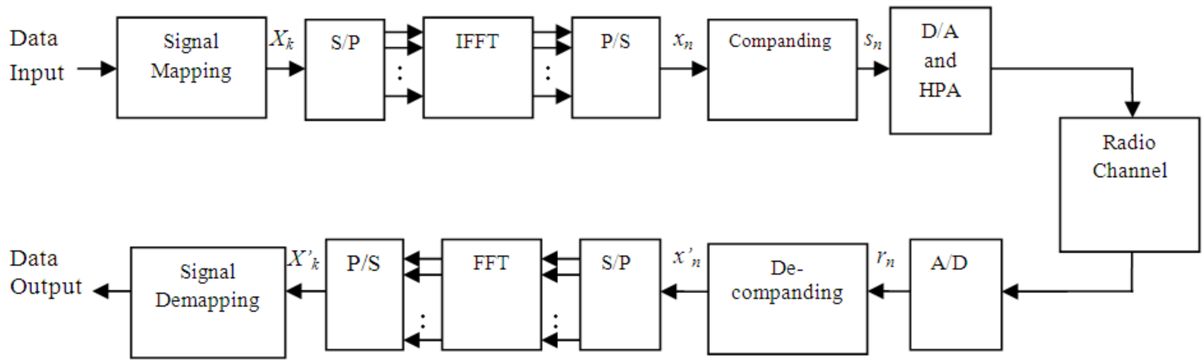

Fig.1.OFDM system using companding techniques.

II. PAPR Problem of the OFDM System

i) Series and Parallel Concepts: In OFDM system design, the series and parallel converter is considered to realize the concept of parallel data transmission.

ii) Series: In a conventional serial data system, the symbols are transmitted sequentially, with the frequency spectrum of each data symbol allowed to occupy the entire available bandwidth. When the data rate is sufficient high, several adjacent symbols may be completely distorted over frequency selective fading or multipath delay spread channel.

iii) Parallel: The spectrum of an individual data element normally occupies only a small part of available bandwidth. Because of dividing an entire channel bandwidth into many narrow sub bands, the frequency response over each individual sub channel is relatively flat. A parallel data transmission system offers possibilities for alleviating this problem encountered with serial systems. Resistance to frequency selective fading.

iv) Modulation/Mapping: The process of mapping the information bits onto the signal constellation plays a fundamental role in determining the properties of the modulation. An OFDM signal consists of a sum of sub-carriers, each of which contains M-ary phase shift keyed (PSK) or quadrature amplitude modulated (QAM) signals.

Figure 1 shows the block diagram of the OFDM system using companding techniques for PAPR reduction. Let K denote the number of sub-carriers used for parallel information transmission and $X_k(0 \le K \le K-1)$ is the k-th complex modulated symbol. Then, x_n is the IFFT output signal and can be represented as

$$x_n = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} X_k . \exp(\frac{j2\pi nk}{K}). \quad (1)$$

The power of OFDM signal x_n can be expressed as

$$|x_n|^2 = \frac{1}{N} \sum_{m=0}^{K-1} \sum_{k=0}^{K-1} X_m X_k \exp(\frac{j2\pi(m-k)n}{K}), \quad (2)$$

The PAPR is calculated as the ratio of maximum power of a signal to its average power.

$$PAPR = \frac{\max(|X_n|^2)}{E(|X_n|^2)} \quad (3)$$

where $(|X_n|^2)$ corresponds to conjugate operator.

$PAPR(dB) = 10\log_{10}(PAPR).$

The PAPR for the OFDM signals within a symbol frame can be defined as

$$PAPR = 10\log_{10} \frac{Max\{|x_n|^2\}}{E\{|x_n|^2\}} (dB), \quad (4)$$

The Cumulative Distribution Function (CDF) is one of the most regularly used parameters, which is used to measure the efficiency of any PAPR technique. Normally, the Complementary CDF (CCDF) is used instead of CDF, which helps us to measure the probability that the PAPR of a certain data block exceeds the given threshold. By implementing the Central Limit Theorem for a multicarrier signal with a large number of sub-carriers, the real and imaginary part of the time- domain signals have a mean of zero and a variance of 0.5 and follow a Gaussian distribution. So Rayleigh distribution is followed for the amplitude of the multi – carrier signal, where as a central chi-square distribution with two degrees of freedom is followed for the power distribution of the system. The CDF of the amplitude of a signal sample is given by

$$F(z) = 1 - \exp(z) \quad (5)$$

The CCDF of the PAPR of the data block is desired is our case to compare outputs of various reduction techniques. This is given by

$$\begin{aligned} P(PAPR > z) &= 1 - P(PAPR \le z) \\ &= 1 - F(z)^N \\ &= 1 - (1 - \exp(-z))^N \end{aligned} \quad (6)$$

The cumulative distribution function (CDF) as following ,

$$F_Y(y) = \frac{1}{2}(1 + erf(y/\sqrt{2\sigma^2}) \quad (7)$$

where z is a given threshold value. The OFDM signals are transformed by the companding function.

The uniformly-distributed companding scheme [18] transforms the OFDM signal into the uniform distribution in the interval, $[0, h_u]$ $(h_u > 0)$. A random variable z is considered to be the amplitude of the companded signal,

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$|S_n|$. Since z is designed to be the uniform distribution, the CDF for the uniform distribution, the CDF for the referred scheme can be represented as

$$F_z^u(z) = \frac{z}{2h_u} + \frac{1}{2}, \quad (8)$$

Because $F_Y(y)$ and $F_z^u(z)$ are strictly monotonically increasing functions, there are homologous inverse functions for both functions. The homologous inverse function of $F_z^u(z)$ can be represented as

$$F_z^{u^{-1}}(z) = 2h_u(z)\left(z - \frac{1}{2}\right). \quad (9)$$

Then, the relationship between $F_Y(y)$ and $F_z^u(z)$ can be represented as

$$\begin{aligned} F_Y(y) &= \Pr\{Y \le y\} \\ &= \Pr\{C_u(Y) \le C_u(y)\} \\ &= F_z^u(C_u(y)). \end{aligned} \quad (10)$$

Solving (10) yields the companding function of the referred scheme,

$$C_u(y) = F_z^{u^{-1}}(F_Y(y)). \quad (11)$$

Substituting (5) and (9) into (11), the companding function of the referred scheme can be written as

$$C_u(x_n) = h_u erf\left(\frac{y}{\sqrt{2\sigma^2}}\right).\exp(j\arg(x_n)), \quad (12)$$

III. Proposed Companding Technique

The firstly derives the general formulas for the OFDM signal with trapezium distribution and then the attenuation effect of the proposed companding scheme are analysed. The distribution of the original OFDM signal transforms into the trapezium distribution in the interval $[0, h_p]$ $(h_p > 0)$. The trapezium distribution of the proposed scheme is shown in figure 2.

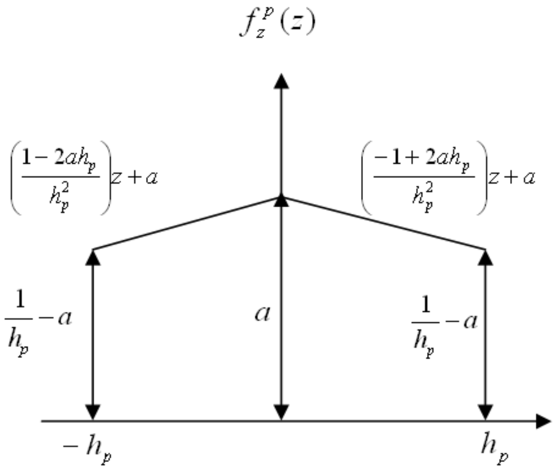

Fig.2. Trapezium distribution of the proposed scheme

The parameter, a, is used to specify the forms of the trapezium distribution, where $0 \le a \le 1/h_p$. The CDF of the proposed scheme given as

$$F_z^p(z) = \frac{z(2ah_p(h_p - z) + z)}{2h_p^2} + \frac{1}{2}, \quad 0 \le z < h_p. \quad (13)$$

The relationship between $F_Y(y)$ and $F_z^p(z)$ can be represented as

$$\begin{aligned} F_Y(y) &= \Pr\{Y \le y\} \\ &= \Pr\{C_p(Y) \le C_p(y)\} \\ &= F_z^p(C_p(y)). \end{aligned} \quad (14)$$

Solving (14) yields the companding function of the proposed scheme (15).

$$C_p(y) = F_z^{p^{-1}}(F_Y(y)). \quad (15)$$

Substituting (5) and (13) into (15), the companding function of the proposed scheme can be expressed as

$$C_p(x_n) = \frac{\sqrt{h_p^2(a^2h_p^2 + (1-2ah_p)erf(\frac{y}{\sqrt{2\sigma^2}})) -}}{1-2ah_p}.ah_p^2.\exp(i\arg(r_n)). \quad (16)$$

The companding function has an inverse transform function and restricted to monotonically increasing function, becomes decompanding function. Thus, the decompanding function of the proposed scheme is given as

$$C_p^{-1}(r_n) = \sqrt{2\sigma^2}.inverf\left(\frac{(ah_p^2 + (1-2ah_p)y')^2 - a^2h_p^4}{h_p^2(1-2ah_p)}\right).\exp(j\arg(r_n)). \quad (17)$$

To make the power of the companded signal the same as that of the original OFDM signal.the power of the companded signal can be represented as

$$\int_{-h_p}^{h_p} z^2 f_z^p(z)dz = \sigma^2. \quad (18)$$

h_p is given as

$$h_p = \sqrt{\frac{6\sigma^2}{(3-2q)}}. \quad (19)$$

Substituting (19) and $a = q/\ h_p$ into (16) and (17)

$$C_p(x_n) = \frac{\sqrt{6}}{1-2q}\left(\sqrt{\frac{\sigma^2}{3-2q}\left(q^2 + (1-2q^2)erf\left(\frac{y}{\sqrt{2\sigma^2}}\right)\right)} - q\sqrt{\frac{\sigma^2}{3-2q}}\right).\exp(j\arg(x_n)), \quad (20)$$

$$C_p^{-1}(r_n) = \sqrt{2\sigma^2}\,inverf\left(\frac{(3-q)y'\left((1-2q)y' + 2q\sqrt{\frac{6\sigma^2}{3-2q}}\right)}{6\sigma^2}\right).\exp(j\arg(r_n)). \quad (21)$$

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The companded signal [14], that composed of an attenuated signal component and companding noise v_n can be represented as

$$s_n = \alpha x_n + v_n, \tag{22}$$

α is given by

$$\alpha = \frac{1}{2\sigma^2}\int_0^{\infty} yh(y) f_{|x_n|}(y)\,dy\ . \tag{23}$$

The OFDM system using companding schemes through AWGN and multipath channels as shown above work. The effect caused by the channel distortion after compensating, the received signal can be expressed as $r_n = s_n + \varepsilon_n$. The recovered signal by the decompanding function can be represented as [11]

$$x'_n = \alpha' r_n + v'_n$$

$$= \frac{s_n + \varepsilon_n}{\alpha} - \frac{v_n}{\alpha} = x_n + \frac{\varepsilon_n}{\alpha}. \tag{24}$$

Equation (24) shows that the decompanding function increases the channel noise ε_n to ε_n/α. In the PAPR of OFDM signal with nonlinear companding scheme [15], if the decompanding function is not used, the equivalent noise is composed of the companding noise v_n and the channel noise ε_n. Then, ε_n/α and $\varepsilon_n + v_n$ is the equivalent noise with and without the decompanding function represented.

IV. Criteria for Selection of PAPR Reduction Technique

PAPR reduction capability:

This is the most important factor in choosing a PAPR reduction technique. Some techniques shows result in other harmful effects. For example, the amplitude clipping technique removes the time domain signal peaks, but results in in-band distortion and out-of-band radiation.

Power increase in transmit signal:

Some techniques require a power increase in the transmit signal after using PAPR reduction techniques. For example, Tone Reservation (TR) requires more signal power because for the PRCs. Tone Injection (TI) uses a set of equivalent constellation points for an original constellation some of its power must be used point to reduce PAPR. The all equivalent constellation points original constellation point, the transmit signal will have more power after applying TI. When the transmit signal power should be equal to or less than that before using a PAPR reduction technique, the transmit signal should be normalized back to the original power level, resulting in BER performance degradation for these techniques.

BER increase at the receiver:

Some techniques may have an increase in BER at the receiver if the transmit signal power is fixed or require larger transmit signal power to maintain the BER after applying the PAPR reduction technique. For example, the BER after applying ACE will be degraded if the transmit signal power is fixed. In SLM, PTS, and interleaving, the entire data block may be lost if the side information is received in error and increase the BER at the receiver.

Loss in data rate:

Some techniques require the data rate to be reduced. In block coding technique requires one out of four information symbols to be dedicated to controlling PAPR. In SLM, PTS, and interleaving, the data rate is reduced due to the side information used to inform the receiver in the transmitter. In these techniques the side information may be received in error unless some form of protection such as channel coding is produced. When channel coding is used, the loss in data rate due to side information is increased.

Computational complexity:

The techniques such as PTS find a solution for the PAPR reduced signal by using much iteration. The PAPR reduction capability of the interleaving technique is better for a more number of inter leavers. Generally, more complex techniques have better PAPR reduction capability.

Other considerations:

The several PAPR reduction techniques do not consider the effect of the components in the transmitter such as the transmit filter, digital-to-analog (D/A) converter, and transmit power amplifier. The PAPR reduction techniques can be used only after performance and cost analyses for realistic environments.

V. Conclusion

Companding technique is an effective technique in reducing high PAPR of the OFDM system. The uniformly distributed companding scheme and the piecewise companding scheme to provide efficient PAPR reduction with a low BER. Furthermore the distribution of the OFDM signal is transformed into the trapezium distribution and the general formulas for the proposed scheme are derived. It is observed that the PAPR is reduced with this technique over theoretical analysis of PAPR. Therefore, the proposed scheme provides a good trade-off between the PAPR reduction and the BER.

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